CHARACTERISTICS OF HIGH-DENSITY CHARGE CLUSTERS: A THEORETICAL MODEL

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ABSTRACT

The self-confined equilibrium properties of a moving EV are investigated and a necessary condition or criteria for which the EV could exist is deduced by macroscopic electron plasma fluid description. We conclude that an EV is a toroidal electron vortex and could exist at various combinations of the electron density, the directional velocity, and the size of the EV in accordance with the derived criteria. The electromagnetic field strength, kinetic and electrostatic potential energy of the EV are also computed with respect to the stated assumptions. A close agreement with experimental data is presented.

A. INTRODUCTION

A highly organized, micron-sized cluster of electrons having soliton behavior, along with number densities equal to Avagadro's number, have been investigated by Kenneth Shoulders since 1980 [1,2,3]. A short Latin acronym has been adopted as suggested by Shoulders, and the collective plasma state of self-contained negatively charged bundle of electrons is called EV, for *strong electrons.*

An explosive electron emission has been shown to involve not individual electrons but electron bunches or avalanches, as also recently described by G.T. Mesyats [4]. The name of "ectons" was adopted by Mesyats to describe these charged particle avalanches. The important arguments for the existence of the entity (EV in the following paragraphs) are cathode craters, sharp oscillations of the explosive electron emission current density with period of 10^{-9} - 10^{-8} s, discontinuity of the glow in the cathode spots of the vacuum sparks and arc, etc. [4].

The EVs have been reported to be generally spherical with diameters on the order of 1.0 to 10 μm, but may be toric, to travel at speeds on the order of 0.1 c, to have electron densities approaching that of a solid, ~6.6 x 10^{29} m^{-3}, with the total number of electrons in a 1 μm diameter EV is ~10^{11}, and with negligible ion content of about one ion per 10^6 electrons. EVs tend to propagate in straight lines for distance of 1.0 to 100 mm and to exhibit a tendency to form other quasi-stable structures, such as linking up like beads in a necklace, etc. [2].

Ziolkowski and Tippett [5] examined theoretically the possibility of the existence of an essentially single-species plasma state represented by a stable packet of charged particles moving collectively through space-time. It is, however, not clear physically how the localized electromagnetic fields, excited by a very short initiation process, provide the mechanism which can overcome the Coulomb force and lead to the possible existence of the collective state represented by the "free"-electron cluster. Beckmann [6] studied the organizational properties of electron clusters theoretically based on the force of the oscillating Faraday field surrounding moving electrons. We know from the electrodynamics, however, that the electric field established by a moving point charge is composed of an electrostatic part (Coulomb field) and voltaic part (Faraday field). The voltaic part is produced by the acceleration of the charge. The two parts of the field will never be balanced with each other. In addition, the physical nature of the EV is collective and cannot be treated with single particle model.

We present here some preliminary estimations on some parameters of the EV as a self-contained, non-neutral plasma; on the electric and magnetic fields established by the EV; on the kinetic and potential energy; on the possible mechanism which overcomes the strong Coulomb force and leads to the existence of the electron cluster; and on the electromagnetic radiation from the EV.

B. ESTIMATIONS OF SOME OF THE PARAMETERS OF AN EV AS A SELF-CONTAINED ELECTRON PLASMA

Following K. Shoulders' data, we assume in the following treatment that the dimension of the EV is $d = 1.0\ \mu m$, i.e. in the spherical model the radius is $r_0 = 0.5\ \mu m$; the electron densities in the EV is $n = 6.6 \times 10^{29}\ m^{-3}$ [3]. In this case the total number of electrons in the spherical EV is

$$N_e = (4/3)\,\pi\, r_0^3\, n = 3.5 \times 10^{11}\ per\ \mathrm{EV}, \tag{1}$$

the total charge is

$$q = N_e\, e = -5.6 \times 10^{-8}\ Coulomb\ per\ \mathrm{EV}, \tag{2}$$

and the mean electron distance is

$$a_e = 2(3/4\pi n)^{1/3} = 1.4 \times 10^{-4}\ \mu m = 1.4\ \mathring{A}. \tag{3}$$

Meanwhile, we take the velocity of the EV, as $v = 0.1\ c = 3 \times 10^7\ m/sec$. This corresponds to the electron kinetic energy of ~ 2.5 KeV, or the EV-creating pulse voltage ~ 2.5 KV.

In this case the electron plasma frequency in the EV is

$$\omega_{pe} = (ne^2/\epsilon_0 m_e)^{1/2} = 56.4\ n^{1/2} = 4.6 \times 10^{16}\ \mathrm{rad/sec.}, \tag{4}$$

where m_e is electron mass and ϵ_0 is permittivity of free space. The ω_{pe} is a measure of space-charge force (or defocusing) in the electron plasma. The characteristic time scale of any change in the EV, therefore, is $1/\omega_{pe} \sim 10^{-17}\ s$.

The electron gyrofrequency on the surface of the EV in the magnetic field (see section C) established by the moving EV is

$$\omega_{ce} = eB/m_e = 1.76 \times 10^{11} B\ rad/sec \sim 1.2 \times 10^{17}\ rad/sec\ . \tag{5}$$

In the self-confinement of an electron cluster, the ω_{ce} is a measure of the self-focusing magnetic force on an electron fluid element.

The electron gyroradius is

$$\rho_e = v/\omega_{ce} = 2.38 \times 10^{-6}\ T^{1/2}\ (eV)\ B^{-1}\ m\ , \tag{6}$$

where T_e is the electron temperature. We assume two temperatures: $T_e = 10\ eV$ and $T_e = 2.5\ KeV$. The $T_e = 10\ eV$ is initial temperature at which the EVs are formed, and possibly the temperature does not change much during the life time of the EV. The $T_e = 2.5\ KeV$ is an assumed final temperatures in which we suppose that the thermal equilibrium in the EV is reached instantaneously with the increase of the directional kinetic energy of the electrons. Substituting both values for T_e in Eq. (6), we have

$$\begin{aligned} \rho_e &= 1.1 \times 10^{-5}\ \mu m \qquad (T_e = 10\ eV)\ , \\ &= 1.7 \times 10^{-4}\ \mu m \qquad (T_e = 2.5\ \mathrm{K}eV)\ . \end{aligned} \tag{6a}$$

The electron *DeBroglie length* in the EV is

$$\lambda\!\!\!^{-} = \hbar/m_e k_B T_e^{-1/2} = 2.76 \times 10^{-10}\, T_e^{-1/2}(eV)\ m, \tag{7}$$

where $\hbar$ is Planck constant ($h/2\pi$), k_B is Boltzman constant. Substituting the T_e to Eq. (7) and comparing the resultant $\lambda\!\!\!^{-}$ with the mean electron distance in the EV, we have

$$\begin{aligned} \lambda\!\!\!^{-} &= 0.87\ \mathring{A} \quad \leq a_e \qquad (T_e = 10\ eV), \\ &= 5.5 \times 10^{-2}\ \mathring{A} \quad \ll a_e \qquad (T_e = 2.5\ KeV). \end{aligned} \tag{7a}$$

This means that the quantum effect needs to be considered in the low temperature situation, but at higher temperatures ($T_e > 100\ eV$) the quantum effect can be neglected.

The thermal kinetic pressure corresponding to these temperatures in the EV are respectively

$$\begin{aligned} p_e = nk_B T_e &= 1.0 \times 10^{12}\ Pascal = 1.0 \times 10^{7}\ atm\ \ pressure, \quad (T_e = 10\ eV), \\ &= 2.6 \times 10^{14}\ Pascal = 2.6 \times 10^{9}\ atm\ pressure, \quad (T_e = 2.5\ KeV). \end{aligned} \tag{8}$$

The electron collision rate [7] in the EV may be estimated by

$$\nu_e = \frac{n e^4 \ln\Lambda}{16\sqrt{\pi}\,\epsilon_0^2 m_e^{1/2} T^{3/2}} = 2.91 \times 10^{-12}\, n \ln\Lambda\, T_e^{-3/2}(eV)\ \sec^{-1}. \tag{9}$$

Here we assumed the electron velocity distribution in the EV is Maxwellian, and $\ln\Lambda$ is the Coulomb logarithm. Substituting the n and T_e and comparing the ν_e with the electron plasma frequency ω_{pe}, we have

$$\begin{aligned} \nu_e &= 1.2 \times 10^{18}\ /Sec \quad >> \omega_{pe} \qquad (T_e = 10\ eV), \\ &= 3.1 \times 10^{14}\ /Sec \quad << \omega_{pe} \qquad (T_e = 2.5\ KeV), \end{aligned} \tag{10}$$

where we assumed $\ln\Lambda = 20$. This means that at the $T_e = 10\ eV$ the electron plasma in the EV is collision dominant, but at $T_e = 2.5\ KeV$ it behaves like a collisionless plasma.

The energy relaxation time of the electron in the EV, the $\tau_e^{(E)}$, may be roughly approximated by $1/\nu_e$:

$$\tau_e^{(E)} \sim 1/\nu_e\,. \tag{11}$$

Then from Eq. (9) we have

$$\begin{aligned} \tau_e^{(E)} &\sim 8.2 \times 10^{-19}\ sec \qquad (T_e = 10\ eV), \\ &\sim 3.2 \times 10^{-15}\ sec \qquad (T_e = 2.5\ KeV), \end{aligned}$$

i.e., both values are far less than the life time of the EV (on the order of a nanosecond, $10^{-9}\,sec$). This means the kinetic energy attained by the electron in the EV from the external field will be quickly thermalized and the electron plasma in the EV can be seen as near Maxwellian.

The coupling parameter Γ of a plasma defined by the ratio of nearest-neighbor Coulomb energy ($e^2/4\pi\epsilon_0 a_e$) to the characteristic thermal energy of a particle ($k_B T_e$) is

$$\Gamma \equiv \frac{e^2}{4\pi\epsilon_0 k_B T_e a_e} = 1.86\times10^{-9} n^{1/3} T_e^{-1}\,(eV)\,. \tag{12}$$

For the EV we have

$$\begin{aligned}\Gamma &\sim 1.5 \qquad (T_e = 10\ eV),\\ &\sim 6\ \mathrm{x}10^{-3} << 1 \qquad (T_e = 2.5\ KeV),\end{aligned} \tag{13}$$

i.e., at $T_e = 10\ eV$ the electron plasma in the EV is strongly coupled and at $T_e = 2.5\ KeV$ it is weakly coupled.

In brief, we see that with $T_e = 10\ eV$ the electron plasma in the EV is a strongly coupled, collision-dominant quantum plasma, and with $T_e = 2.5\ KeV$ it is a weakly coupled collisionless classical plasma.

In the following we regard the EV as a weakly coupled collisionless classical plasma and treat the EV using classical electrodynamics and plasma fluid descriptions.

C. THE ELECTRIC AND MAGNETIC FIELDS

We consider a spherical model of the EV. In view of the spherical symmetry, in this case the electromagnetic fields of the EV can be approximated by a moving point charge model. From the well-known general equations of the fields established by a moving point charge [8], the electric and magnetic field of the moving EV may be expressed by

$$\boldsymbol{E} = \frac{q}{4\pi\epsilon_0 r_0^3}\left[r\,\boldsymbol{e}_r + \frac{3}{c}r\Big((\boldsymbol{v}\cdot\boldsymbol{e}_r)\,\boldsymbol{e}_r - \boldsymbol{v}\Big) + \frac{r^2}{c^2}\,\boldsymbol{e}_r\times(\boldsymbol{e}_r\times\dot{\boldsymbol{v}})\right],\ (r<r_0) \tag{14}$$

$$\boldsymbol{B} = \frac{q}{4\pi\epsilon_0 r_0^3}\left[\frac{r}{c^2}\,\boldsymbol{v}\times\boldsymbol{e}_r + \frac{r^2}{c^3}\,\dot{\boldsymbol{v}}\times\boldsymbol{e}_r\right],\ (r<r_0) \tag{15}$$

and

$$\boldsymbol{E} = \frac{q}{4\pi\epsilon_0}\left[\frac{\boldsymbol{e}_r}{\boldsymbol{r}^2} + \frac{3(\boldsymbol{v}\cdot\boldsymbol{e}_r)\,\boldsymbol{e}_r - \boldsymbol{v}}{c\,r^2} + \frac{\boldsymbol{e}_r\times(\boldsymbol{e}_r\times\dot{\boldsymbol{v}})}{c^2 r}\right],\quad (r>r_0) \tag{16}$$

$$\boldsymbol{B} = \frac{q}{4\pi\epsilon_0}\left[\frac{\boldsymbol{v}\times\boldsymbol{e}_r}{c^2 r^2} + \frac{\dot{\boldsymbol{v}}\times\boldsymbol{e}_r}{c^3 r}\right],\qquad (r>r_0)\,, \tag{17}$$

where $r^2 = x^2 + y^2 + z^2$, $\boldsymbol{e}_r = \mathbf{r}/|\mathbf{r}|$ is unit vector, r_0 is the radius of the spherical EV, $\dot{\boldsymbol{v}} = \partial\,\boldsymbol{v}/\partial t$ is the acceleration, and uniform electron charge distribution in the EV is assumed. We see clearly that the electric fields are composed of both electrostatic and voltaic parts, and usually the latter is much smaller than former.

For simplicity, we assume the EV moves along the straight line, axis Z-direction, then the above equations can be reduced to

$$\boldsymbol{E} = \frac{q}{4\pi\epsilon_0 r_0^3}\left[\left((1+\frac{3v}{c}\cos\theta)\,r + \frac{\dot{v}r^2}{c^2}\cos\theta\right)\boldsymbol{e}_r - \left(\frac{3v}{c}r + \frac{\dot{v}}{c^2}r^2\right)\boldsymbol{e}_z\right],\qquad (r<r_0) \tag{18}$$

$$\boldsymbol{B} = \frac{q\,v\,r}{4\pi\epsilon_0 c^2 r_0^3}\left[1 + \frac{\dot{v}}{c\,v}r\right]\sin\theta\ \boldsymbol{e}_z\times\boldsymbol{e}_r,\qquad (r<r_0) \tag{19}$$

$$\Xi = \frac{q}{4\pi\epsilon_0 r^2}\left[(1+\frac{3v}{c}\cos\theta+\frac{\dot{v}r}{c^2}\cos\theta)\,\boldsymbol{e}_r-(\frac{3v}{c}+\frac{\dot{v}r}{c^2})\,\boldsymbol{e}_z\right], \qquad (r>r_0 \tag{20}$$

$$\boldsymbol{B} = \frac{qv}{4\pi\epsilon_0 c^2 r^2}\left[1+\frac{\dot{v}r}{cv}\right]\sin\theta\ \boldsymbol{e}_z\times\boldsymbol{e}_r, \qquad (r>r_0) \tag{21}$$

where θ is the angle between ***r*** and *Z*-axis (in the direction of *v*).

As a typical case, we now assume that the distance between an EV launcher and the target is *l* = 10.0 mm and the applied pulse voltage is V = 2.5 *KV*. Then the acceleration of the EV is

$$\dot{v} = eV/m_e l = 4.4 \text{ x } 10^{16}\ m/sec^2. \tag{22}$$

In this case we have $\dot{v}r/c^2 \sim \dot{v}r/cv \lesssim 10^{-7} \ll 1$ in the near surface and inside of the EV, i.e. the effect of acceleration on the ***E*** and ***B*** fields are negligible and Eqs. (18) - (21) can be reduced to

$$\boldsymbol{E} = \frac{qr}{4\pi\epsilon_0 r_0^3}\left[(1+\frac{3v}{c}\cos\theta)\,\boldsymbol{e}_r-\frac{3v}{c}\boldsymbol{e}_z\right], \qquad (r<r_0) \tag{23}$$

$$\boldsymbol{B} = \frac{qvr}{4\pi\epsilon_0 c^2 r_0^3}\sin\theta\ \boldsymbol{e}_z\times\boldsymbol{e}_r, \qquad (r<r_0) \tag{24}$$

and

$$\boldsymbol{E} = \frac{q}{4\pi\epsilon_0 r^2}\left[(1+\frac{3v}{c}\cos\theta)\,\boldsymbol{e}_r-\frac{3v}{c}\boldsymbol{e}_z\right], \qquad (r>r_0) \tag{25}$$

$$\boldsymbol{B} = \frac{qv}{4\pi\epsilon_0 c^2 r^2}\sin\theta\ \boldsymbol{e}_z\times\boldsymbol{e}_r, \qquad (r>r_0). \tag{26}$$

The fields on the surface of the EV are given by

$$\boldsymbol{E} = \frac{q}{4\pi\epsilon_0 r_0^2}\left[(1+\frac{3v}{c}\cos\theta)\,\boldsymbol{e}_r-\frac{3v}{c}\boldsymbol{e}_z\right], \tag{27}$$

$$\boldsymbol{B} = \frac{qv}{4\pi\epsilon_0 c^2 r_0^2}\sin\theta\ \boldsymbol{e}_z\times\boldsymbol{e}_r\ . \tag{28}$$

Substituting r_0 = 0.5 μm, *v* = 0.1 c. and Eq. (2) for *q* we have

$$\boldsymbol{E} = -2.0\times10^{15}\left[(1+0.3\cos\theta)\,\boldsymbol{e}_r-\frac{3v}{c}\boldsymbol{e}_z\right]V/m, \qquad (r=r_0) \tag{29}$$

$$\boldsymbol{B} = 6.7\times10^5\sin\theta\ \boldsymbol{e}_r\times\boldsymbol{e}_z\ Tesla. \qquad (r=r_0) \tag{30}$$

We see that the local fields established by the EV are very strong and beyond any present laboratory upper limit. The magnetic field (~10^6 *Tesla)* of the EV exceeds the highest laboratory field (~10^4 *Tesla)* produced by explosive techniques and approaches the field strength on the surface of some neutron star (~10^7 - 10^8 *Tesla)*. The super strong electric fields on the EV will ionize any materials on its path.

D. KINETIC ENERGY

The kinetic energy of an electron EV is

$$W_{EV} = N_e eV = 8.8 \text{ x } 10^{14}\ eV = 1.4 \text{ x } 10^{-4} J\ , \tag{31}$$

where the applied voltage $V = 2.5\ KV$ is assumed.

For a nuclear EV (capable of causing a nuclear reaction) where the ion number $N_i \sim 10^{-6} N_e$, the kinetic energy is almost the same as the electron EV without positive ions:

$$W_{NEV} = (N_e + Z N_i)\ eV \simeq N_e eV = W_{EV},\ \ N_i / N_e \sim 10^{-6} \ll 1\ , \tag{32}$$

where Z is atomic number of the ion. The ions in the EV will be attracted or drawn by the overwhelming majority of electrons in the EV and attain the same velocity as electrons. The kinetic energy attained by the ion in the EV then is

$$W_{EV}^{(i)} = \frac{1}{2} M_i v_e^2 = \frac{M_i}{m_e} (\frac{1}{2} m_e v_e^2) = \frac{M_i}{m_e} eV \tag{33}$$

where m_e, M_i is the mass of electron and the ion, v_e is the electron velocity attained due to the applied voltage V. The energy attained by a single ion using the same voltage is

$$W^{(i)} = Z\,eV \tag{34}$$

Comparing Eqs. (33) and (34) we have

$$W_{EV}^{(i)} / W^{(i)} = \frac{M_i}{Z m_e} = \frac{m_p M_i}{Z m_e m_p} = 1836 \frac{A}{Z} \tag{35}$$

where A is atomic weight of the ion and m_p is proton mass.

This means the EV could act as a relatively simple mini-accelerator for accelerating positive ions. For example, in the $V = 2.5\ KV$ applied voltage, a proton (deuteron) will attain 2.5 *KeV* energy. However, the proton (deuteron) in the EV could get 4.6 *MeV* (9.2 *MeV*)! This kinetic energy imparted to the positive ions is now sufficient to overcome the Coulomb barrier of the nucleus and produce nuclear reactions. When a large number of EVs are produced and strike a target, the nuclear reactions rate can be quite high. Shoulders et al. [3] and Hal Fox et al. [9] have proposed this nuclear acceleration mechanism as a possible explanation of the anomalous nuclear transmutation phenomena [10, 11, 12].

E. ELECTROSTATIC POTENTIAL ENERGY

How much energy is required to produce an electron cluster - EV? Also, what is the electrostatic potential energy of the EV? We consider a spherical EV with uniform electron density n. From Eqs. (23), (25) the electrostatic field established by the EV is

$$\mathbf{E} = \frac{q\,r}{4\pi\epsilon_0 r_0^3}\,\mathbf{e}_r = \frac{n\,e\,r}{3\epsilon_0}\,\mathbf{e}_r, \qquad (r < r_0) \tag{36a}$$

$$= \frac{q}{4\pi\epsilon_0 r^2}\,\mathbf{e}_r = \frac{n e r_0^3}{3\epsilon_0 r^2}\,\mathbf{e}_r\,, \qquad (r > r_0)\,. \tag{36b}$$

The potential is

$$V = -\int_\infty^r \mathbf{E}\cdot d\mathbf{r} = \frac{n e r_0^2}{2\epsilon_0} - \frac{n e r^2}{6\epsilon_0}\,, \qquad (r < r_0) \tag{37a}$$

$$= \frac{n e r_0^3}{3\epsilon_0 r}\,, \qquad (r > r_0)\,. \tag{37b}$$

Thus the potential difference from center to surface of the EV is

$$\Delta V = V(r=0) - V(r=r_0) = \frac{n e r_0^2}{6\epsilon_0} = -5.0\times 10^8\,V \tag{38}$$

with $r_0 = 0.5$ μm and $n = 6.6$ x 10^{29}/ m^3.

The electrostatic potential energy is

$$W_{ES} = \frac{1}{2}\iiint \rho\, V\, d\tau = \frac{1}{2}\int_0^{2\pi}\int_0^{\pi}\int_0^{r_0} n^2 e^2 \left(\frac{r_0^2}{2\epsilon_0} - \frac{r^2}{6\epsilon_0}\right) r^2 \sin\theta\, dr\, d\theta\, d\phi$$

$$= \frac{4\pi n^2 e^2 r_0^5}{15\epsilon_0} = \frac{3 q^2}{20\pi\epsilon_0 r_0}\,. \tag{39}$$

Substituting the given data of r_o and n to Eq. (39), we have

$$W_{ES} = 33.0 \text{ J} = 2.1 \times 10^{14}\, MeV.$$

Or the corresponding potential energy density is

$$w_{ES} = W_{ES} / \frac{4\pi}{3}\pi r_0^3 = \frac{9 q^2}{80\pi^2\epsilon_0 r_0^4} = 6.4\times 10^{19}\, J/m^3\,. \tag{40}$$

This is an incredible, unimaginable energy combination. If the EV were "blasted" and this potential energy were completely transformed to kinetic energy, then each electron in the EV would have

$$W_e = W_{ES} / N_e = n\, e^2\, r_0^2 / 5\, \epsilon_0 = 9.6 \text{ x } 10^{-11}\, J = 6.0 \text{ x } 10^2 MeV \tag{41}$$

which corresponding to electron temperature ~ 10^{12} K. That would be ~10^2 to 10^3 times higher than the temperature in the core of the hydrogen bomb explosion, a supernova explosion or in a "white dwarf."

For a nuclear EV in which $N_i/N_e \sim 10^{-6}$, the electrostatic potential energy is almost the same as the expression Eq. (39) (assuming ion distribution in the EV is uniform). If the nuclear EV were "blasted" (such as blasting a target anode), each electron would have almost the same amount of energy, as shown by Eq. (41), and each ion would have

$$W_i = M_i\, v_e^2/2 = M_i\, W_e/m_e = 1836\text{ A } W_e = 1.1 \times 10^6 \text{ A } MeV \tag{42}$$

where A is the atomic weight of positive ions and assuming the ions would be carried or drawn by the expanding electrons. Even if the EV with 1 μm diameter expands to only a 10% increase in diameter of the EV (on impacting the target), according to Eq. (39) about 9% of the potential energy will transform to kinetic energy of the electrons and ions, and each ion would receive an average kinetic energy of $W_i \sim 10^5$ A MeV. This may be the main source of the nuclear effects of the EV.

The electrostatic expansion force in the EV, from Eq. (39), could be estimated by

$$f_{ES} = -\frac{\partial W_{ES}}{\partial r} = \frac{3\,q^2}{20\,\pi\epsilon_0\, r^2}\,, \tag{43}$$

and the expansion pressure on the surface is

$$P_{ES} = f_{ES}/4\,\pi\, r_0^2 = \frac{3\,q^2}{80\,\pi^2\epsilon_0\, r_0^4} = \frac{n^2 e^2 r_0^2}{15\,\epsilon_0} \tag{44}$$

$$= 2.1 \times 10^{19} \text{ pascal} = 2.0 \times 10^{14} \text{ atm pressure.}$$

This is higher than any known pressure created in scientific laboratories.

We see from above that, in some respect, the EV can be seen as a mini super-projectile. When it arrives at a target, it will be exploded, and produce nuclear reactions (nuclear EV case), and release a large amount of potential energy to the target.

F. IS EV SPHERICAL OR TOROIDAL?

How can the super strong electric repulsive force in the EV be balanced by another force? And which model is the most likely to explain this phenomena, the spherical or the toroidal model?

To solve the problem, we begin with the non-neutral plasma macroscopic fluid equations [13]:

the *continuity equation*

$$\frac{\partial}{\partial t}\, n + \nabla\cdot(n\,\boldsymbol{u}) = 0 \tag{45}$$

and the *momentum equation* (or force balance equation)

$$n\left(\frac{\partial}{\partial t} + \boldsymbol{u}\cdot\nabla\right)\boldsymbol{p} = -\nabla\cdot\hat{\boldsymbol{P}} + n\,e\,(\boldsymbol{E} + \boldsymbol{u}\times\boldsymbol{B}), \tag{46}$$

where n is electron density, $\boldsymbol{u}$ electron fluid velocity, $\boldsymbol{p}$ mean momentum, and $\hat{\boldsymbol{P}}$ thermal pressure tensor. The Eqs. (44) and (45) are to be supplemented by *Maxwell's equations* for the self-consistent evolution of $\boldsymbol{E}$ and $\boldsymbol{B}$.

$$\nabla \cdot \boldsymbol{E} = \frac{n e}{\epsilon_0}, \qquad \nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}, \tag{47a}$$

$$\nabla \times \boldsymbol{B} = \mu_0 e n \boldsymbol{u} + \frac{1}{c^2}\frac{\partial \boldsymbol{E}}{\partial t}, \qquad \nabla \cdot \boldsymbol{B} = 0 . \tag{47b}$$

We are mainly interested in the possibility of the existence of collective state of the electron cluster. For this task we need an equilibrium solution to the Eqs. (45) - (47). Carrying out an equilibrium analysis of the equations by setting $\partial/\partial t = 0$, we have

$$\nabla \cdot (n_0 \boldsymbol{u_0}) = 0 , \tag{48}$$

$$(\boldsymbol{u_0} \cdot \nabla)\boldsymbol{p_0} = n_0 e (\boldsymbol{E_0} + \boldsymbol{u_0} \times \boldsymbol{B_0}) , \tag{49}$$

and

$$\nabla \cdot \boldsymbol{E_0} = \frac{n_0 e}{\epsilon_0}, \qquad \nabla \times \boldsymbol{E_0} = 0 , \tag{50a}$$

$$\nabla \times \boldsymbol{B_0} = \mu_0 e n_0 \boldsymbol{u_0} , \quad \nabla \cdot \boldsymbol{B_0} = 0 , \tag{50b}$$

where $n_0 = n_0(\mathbf{r})$, $\boldsymbol{u}_0 = \boldsymbol{u}_0(r)$, $\boldsymbol{p}_0 = \boldsymbol{p}_0(r)$, $\boldsymbol{E}_0 = \boldsymbol{E}_0(r)$, *and* $\boldsymbol{B}_0 = \boldsymbol{B}_0(r)$ are the macroscopic equilibrium quantities. Notice that we dropped the thermal pressure-gradient term $\nabla \cdot \hat{\boldsymbol{P}}$ in the force balance equation (46). The reason is that the thermal kinetic pressure (Eq. 8) is negligibly smaller than the electrostatic pressure (Eq. (44)) in the EV. With the equilibrium equations (48) - (50), we now consider the possibilities of equilibrium in the spherical and toroidal model of the EV respectively. For simplicity, we will drop the subscript “0” on the equilibrium quantities in the following equations.

First consider the spherical model of the EV. We can reasonably assume that the fields $\boldsymbol{E}$ and $\boldsymbol{B}$ in the equilibrium equations are given by Eqs. (23) and (24), and the term $(\boldsymbol{u} \cdot \nabla)\boldsymbol{p}$ (a centrifugal force) in Eq. (49) is negligible compared with electric force $ne\boldsymbol{E}$. In this case, Eq. (49) becomes

$$n e (\boldsymbol{E} + \boldsymbol{u} \times \boldsymbol{B}) = 0 . \tag{51}$$

This equation means that the electric repulsive force in the EV is balanced by the magnetic force which is caused by the perpendicular (to magnetic field $\boldsymbol{B}$) motion

$$\boldsymbol{u}_\perp = (\boldsymbol{E} \times \boldsymbol{B}) / B^2 \tag{52}$$

and the parallel motion $\boldsymbol{u}_{//} = \boldsymbol{E} \cdot \boldsymbol{B} / B^2$ of the electron fluid, corresponding to the currents $\boldsymbol{j}_\perp = ne\boldsymbol{u}_\perp$ and $\boldsymbol{j}_{//} = ne\boldsymbol{u}_{//}$ respectively. Notice that the perpendicular motion $\boldsymbol{u}_\perp$ of Eq. (52) is just the $\boldsymbol{E} \times \boldsymbol{B}$ drift velocity of the electron (Fig 1). This result is reasonable. For the EV electron plasma, we have from Eqs. (12), (13), (23), and (24) that

$$|\rho_e \frac{\partial B}{\partial r}| \sim 3\rho_e \frac{B}{r_0} \sim 10^{-3} B \ll B \quad \text{and} \quad |\frac{1}{\omega_c}\frac{\partial B}{\partial t}| \sim \dot{v}\, B/\omega_{ce} v \sim 10^{-9} B \ll B\,.$$

This means that the electron motion in the EV may be approximated by electron guiding center motion. The physical mechanism is also clear. We know that the expansion force caused by the thermal pressure of a plasma in a magnetic field ***B*** can be balanced by the magnetic force caused by the diamagnetic drift motion of the plasma fluid element, $\boldsymbol{u} = -\nabla p \times \boldsymbol{B} / B^2$ ($p = nkT$), which results from the gyromotion of the plasma particle around the magnetic field, as shown in Fig. 1, and the density and/or temperature gradient. In the EV case, the radial electrostatic repulsive force is balanced by the magnetic force caused by the $\boldsymbol{E} \times \boldsymbol{B} / B^2$ drift motion of the electron fluid element.

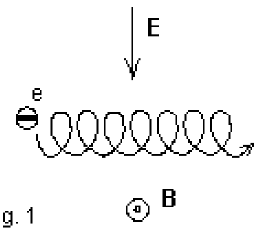

Fig. 1

The electron motion in the crossed electric and magnetic fields. The electron gyrate about the magnetic field lines, accompanied by a drift. The drift velocity is **v** = **E** x **B**/ B^2.

We know from Eq. (24) that the magnetic field in the spherical EV is around the axis *Z*, i.e. around the sphere. The parallel motion of the electron fluid does not change the shape of the EV sphere. However, the perpendicular motion will tend to change the shape of the sphere. Substituting Eqs. (23) and (24) to (52), we have

$$\boldsymbol{u}_{\perp} = \frac{c^2}{v \sin\theta}\left[\left(\frac{3v}{c} \sin^2\theta - \cos\theta \right) \boldsymbol{e}_r + \boldsymbol{e}_z \right]. \tag{53}$$

We see that the $\boldsymbol{u}_\perp$ varies with angle θ, but not along the circle. This process will instantaneously (in about 10^{-16} s) lead to deformation of the spherical cluster to an oblate spheroid and the most likely result will be that the sphere will evolve to a toroidal shape. We will see in the following paragraphs that in the toroidal configuration the electron cluster and electromagnetic field will self-consistently be confined each other and there is no possibility to deform to any other shape which has better equilibrium properties. However, the toroid may disrupt to smaller toroidal groups or become self-destroyed by various instabilities.

Now we consider a toroidial model of the EV. We assume the EV is a toroidal electron plasma loop with minor radius a and major radius R_0 confined by a toroidal magnetic field $\boldsymbol{B_0}$ (see Fig. 2). In the EV the toroidal field actually is established by the moving electron cluster itself. We assume that the diameter of the toroidal EV, the $2R$, is the same value as the diameter of the spherical EV, i.e. $2R = 2r_0 = d = 1\mu$m. In this case, the toroidal magnetic field $\boldsymbol{B_0}$ can be roughly approximated by the magnetic field of the spherical EV, Eq. (28), averaging over the surface of the spherical EV, i.e.

$$|\boldsymbol{B_0}| = \frac{qv}{4\pi\epsilon_0 c^2 r_0^2}\frac{1}{\pi}\int_0^{\pi} \sin\theta\, d\theta = \frac{qv}{2\pi^2\epsilon_0 c^2 r_0^2} = \frac{2ner_0 v}{3\pi\epsilon_0 c^2} = \frac{2neRv}{3\pi\epsilon_0 c^2} = 4.2\times 10^5 T \tag{54}$$

where $R = R_0 + a$.

As an approximation, the toroidal electron loop can be simplified by treating it as an infinitely long ($2\pi R_0 \gg 2\pi a$) cylindrical column of electrons. We further assume that the $\mathbf{B}_0$ is uniform along the axis of the column. We now select a cylindrical coordinate consistent with the column and assume all of the equilibrium parameters in the steady state ($\partial/\partial t = 0$) are axi-symmetric and uniform in the Z-direction, i.e. $n(\boldsymbol{r}) = n(r)\,\boldsymbol{e}_r$, $\boldsymbol{u}(\boldsymbol{r}) = u_\theta(r)\,\boldsymbol{e}_\theta + u_z(r)\,\boldsymbol{e}_z$ and $\partial n/\partial z = \partial \boldsymbol{u}/\partial z = 0$. The azimuthal current $\boldsymbol{J}_\theta(r) = neu_\theta(r)\,\boldsymbol{e}_\theta$ generally induces an axial self-magnetic field $B_Z(r)$ and the axial current $\boldsymbol{J}_z(r) = neu_z(r)\,\boldsymbol{e}_z$ generally induces azimuthal self-magnetic field $B_\theta(r)$. Thus the total magnetic field is $\boldsymbol{B}(r) = B_\theta(r)\,\boldsymbol{e}_\theta + (B_0 + B_z(r))\,\boldsymbol{e}_z$.

The equilibrium equation (49) now becomes a radial force balance equation

$$-\frac{\beta\, m_e u_\theta^2}{r} = e\,[E_r + u_z B_\theta + u_\theta (B_0 + B_z)]\,, \tag{55}$$

where $\beta(r) = (1 - u^2(r)/c^2)^{-1/2}$ and the self-generated fields E_r, B_θ, and B_z are determined from the Maxwell equations

$$\frac{1}{r}\frac{\partial}{\partial r}(r E_r(r)) = \frac{e}{\epsilon_0} n(r)\,, \tag{56a}$$

$$\frac{1}{r}\frac{\partial}{\partial r}(r B_\theta(r)) = \mu_0 e\, n(r)\, u_z(r)\,, \tag{56b}$$

$$\frac{\partial}{\partial r} B_z(r) = -\mu_0 e\, n(r)\, u_\theta(r)\,. \tag{56c}$$

The solutions of the Maxwell Eq. (56) are

$$E(r) = \frac{e}{\epsilon_0 r}\int_0^r n(r)\, r\, dr\,, \tag{57a}$$

$$B_\theta(r) = \frac{\mu_0 e}{r}\int_0^r n(r)\, u_z(r)\, r\, dr\,, \tag{57b}$$

$$B_z(r) = -\mu_0 e \int_r^\infty n(r)\, u_\theta(r)\, dr\,. \tag{57c}$$

Substituting Eqs. (57) to (55), we have [13]

$$\beta(r)\,\omega_{re}^2(r) = \omega_{ce}\,\omega_{re}(r)\left(1 + \frac{\mu_0 e}{B_0}\int_r^\infty n(r)\, r\, \omega_{re}(r)\, dr\right) + \frac{e^2}{\epsilon_0 m_e r^2}\int_0^r n(r)\left(1 - \frac{u_z^2(r)}{c^2}\right) r\, dr \tag{58}$$

where $\omega_{re}(r)$ is the equilibrium angular velocity of an electron fluid element

$$\omega_{re}(r) = u_\theta(r)/r \tag{59}$$

and $\omega_{ce} = eB/m_e$ is the non-relativistic cyclotron frequency (Eq. (12)). We see that the rotation of the electron plasma is a fundamental characteristics for self-confinement of the EV.

The physical meaning of the force balance equation (55) is obvious. The term on the left side of the Eq. (55), $\beta\, m_e u_e^2/r$, is a centrifugal force. Thus, Eq. (55) means that the ombined electrostatic and centrifugal forces on an electron fluid element are balanced by the inward magnetic force. The magnetic force is caused by the diamagnetic drift motion of the electron fluid element, $\boldsymbol{E} \times \boldsymbol{B}/B^2$ drift and $m_e u_\theta^2/r\, \boldsymbol{e}_r \times \boldsymbol{B}/B^2$ drift, i.e. the rotation of the electron fluid around the symmetric axis. Therefore, for a moving toroidal electron cluster, a self-consistent equilibrium is possible, thus, the toroidal EV can exist.

Ken Shoulders [14] has for several years been aware of the probable toroidal nature of high-density electron clusters and has explicitly dealt with the likely nature of their formation.

We consider a simplified case: a non-relativistic electron plasma column with uniform density profile and $u^2/c^2 \ll 1$, $u_z B_0 \ll E_r$ and $B_z \ll B_0$. In this situation the radial force balance equation (58) reduces to

$$\omega_{re}^2 - \omega_{ce}\,\omega_{re} + \frac{1}{2}\,\omega_{pe}^2 = 0\,. \tag{60}$$

Solving the Eq. (60) for ω_{re} gives two equilibrium rotational velocities of the electron column

$$\omega_{re} = \omega_{re}^{\pm} \equiv \tfrac{1}{2}\,\omega_{ce}\left[1 \pm \left(1 - \frac{2\,\omega_{pe}^2}{\omega_{ce}^2}\right)^{1/2}\right] \tag{61}$$

where $\omega^+_{re}(\omega_{re})$ corresponds to a fast (slow) rotation of the electron fluid. In the uniform density profile the rotational velocity ω^+_{re} *or* ω^-_{re} are constant, i.e. the electron fluid will make a rigid rotation around the symmetric axis. Using Eq. (54) we have $\omega_{ce} = eB_0/m_e = 7.4 \times 10^{16}$ rad/sec. Substituting the ω_{ce} and Eq. (4) for ω_{pe} to the Eq. (61) gives $2\omega_{pe}^2/\omega_{ce}^2 = 0.77$, $\omega^+_{re} = 0.74\ \omega_{ce} = 5.5 \times 10^{16}$ rad/sec and $\omega^-_{re} = 0.26\ \omega_{ce} = 1.9 \times 10^{16}$ rad/sec. (With relativistic treatment, we will have $\omega^{\pm}_{re} \sim 10^{13}$ rad/sec.) These are super high speed rotations.

We note especially from Eq. (61) that when $2\omega^2_{pe} / \omega^2_{ce} > 1$, the rotation velocity ω_{re} becomes imaginary, that is the radial equilibrium does not exist. This means a necessary condition for existence of equilibrium EV is

$$2\omega^2_{pr} / \omega^2_{ce} = 2\,n\,m_e / \epsilon_0 B_0^2 \le 1 \tag{62a}$$

or

$$B_0 \ge (2\,n\,m_e / \epsilon_0)^{1/2} \tag{62b}$$

Substituting Eq. (54) for B_0 to the Eq. (62b) will give ***radial equilibrium criteria*** or ***EV criteria***:

$$nd^2v^2 \ge 18\,\pi^2 c^2 m_e / \mu_0 e^2 \ge 4.5 \times 10^{32}\ \text{meter/sec}^2 \tag{63a}$$

or

$$nWd^2 \ge 9\pi^2 c^2 m_e^2 / \mu_0 e^2 \ge 2.1 \times 10^{21}\ \text{meter/sec}^2\,, \tag{63b}$$

where $d = 2R$ is the dimension of the EV, v is directional velocity and W is the corresponding kinetic energy (in units of eV) of the EV electron.

This criteria tells us that the electron density n, directional velocity v (or kinetic energy W) and the size of the EV, the d, are related to each other in comprising a equilibrium EV. Or in other words, the EV could exist at various combinations of n, v (or W) and d in accordance with this criteria.

As a test for the criteria, let's examine K. Shoulders' EV data [2]. With $n = 6.6 \times 10^{29}$ m^{-3} and $v = 0.1$ c $= 3 \times 10^7$ m/sec, the criteria (63) gives d $\ge$ 0.87 μm. This is close to the size range that Shoulders measured. Therefore, agreement between this theory and experimental data is excellent.

According to the criteria, let's estimate the EV size at low directional energy, $W = (10$ to $100)\,eV$ situation. If we assume the electron density is still $n = 6.6 \times 10^{29}$ m^{-3}, then the criteria (63) gives $d \ge$ (13 to 4) μm, corresponding to electron number (5.9×10^{13} to 5.6×10^{12}) in the EV. That is, equilibrium in larger dimension EVs is possible, as long as the EV is formed and launched. However, some of the EVs could be broken up into smaller ones through instabilities. In the double layer, near surface and crack regions of a hydrated metal cathode, a number of large EVs could be produced and may have an important role in nuclear transmutation. A large EV may contain or transport a larger number of positive nuclear ions and thereby the nuclear reactions rate may be increased.

At low electron density, for example $n \sim 3 \times 10^{25} \mathrm{m}^{-3}$ (~ air density), and the directional energy on the order of room temperature energy, $W \sim 0.025$ eV, the criteria gives d ≥ 5 cm. This is the size range of ball-lightning. The ball-lightning may be a huge EV.

Now we consider a more general case where density *n(r)* is not uniform. In this case the Eq. (54) reduces to

$$\omega_{re}^2 - \omega_{ce}\,\omega_{re} + \frac{e^2}{\epsilon_0 m_e r^2}\int_0^r n(r)\,r\,dr = 0\,. \tag{64}$$

The solution of the Eq. (64) is

$$\omega_{re} = \omega_{re}^{\pm}(r) \equiv \frac{1}{2}\,\omega_{ce}\left[1 - \left(1 - \frac{4e^2}{\epsilon_0 m_e \omega_{ce}^2 r^2}\int_0^r n(r)\,r\,dr\right)^{1/2}\right]. \tag{65}$$

We see that, in general, the equilibrium angular velocity of the electron fluid is not constant, but varies with radius of the column, i.e. there exists a shear in the angular velocity profile.

The angular velocity shear has an important effect on the instability of electron plasma. The *diocotron* instability [13, 15] is one of the most ubiquitous instabilities in low density electron plasmas with shear in flow velocity. In the relatively low density region with $\omega_{pe}^2 / \omega_{ce} \ll 1$ in the EV, such instability could be developed. This instability is likely one of the reasons that a large EV may be broken up into several smaller EV's.

In the above, we considered a simplified case, i.e. $u^2 / c^2 \ll 1$, $u_z B_\theta \ll E_r$, and $B_z \ll B_0$. In a more general case, these restrictions should be removed and the toroidal effect must be included. This general case is beyond the scope of this paper.

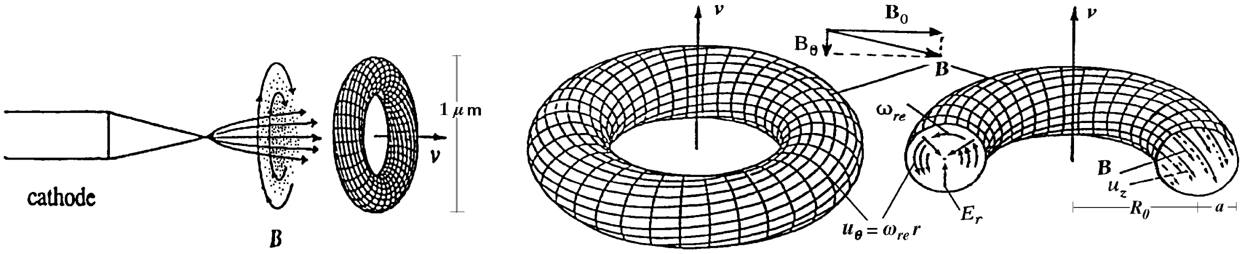

Fig. 2 A Toroidal model of the EV. The combined radial electrostatic force – $neE(r)\mathbf{e}_r$ and the centrifugal force $nm_e u_\theta^2 / r\,\mathbf{e}_r$ on the electron fluid element are balanced by the inward magnetic force $neu_\theta (B_0 + B_z)\,\mathbf{e}_r$ and $neu_z B_\theta \mathbf{e}_r$, through the high speed angular rotation of the element.

According to the above discussion, we imagine a toroidal EV as shown in Fig. 2. The electron plasma is mainly confined by the toroidal magnetic field $\boldsymbol{B}_0$ which is established by the directional motion of the electrons in the applied potential difference. The radial electrostatic force neE_r and centrifugale force $m_e u_\theta^2/r$ on the electron fluid element is balanced by inward magnetic force which is caused by $\boldsymbol{E} \times \boldsymbol{B} / B^2$ drift and $\dfrac{m_e u_\theta^2}{r}\, \boldsymbol{e}_r \times \boldsymbol{B}/B^2$ drift motion of the electron fluid element. Meanwhile, directional acceleration motion of the electron cluster, nonuniform electron density profile, and diamagnetic pressure gradient drift, $-\nabla p \times \boldsymbol{B}/B^2$ etc., may cause the toroidal component of the plasma flow velocity u_z, or current J_z, and poloidal magnetic field B_θ. The inside current J_z could contribute to confine outside electrons, and the outside magnetic field B_θ contribute to confine the inside current. Thus, electron plasma, electric and magnetic fields and currents etc., may self-consistently be formed as a self-sustained, closed-equilibrium system.

G. RADIATION

Electromagnetic radiation from the EV can be expressed approximately by using the accelerated point charge when the observation distance is far larger than the EV size. The energy transported by electromagnetic field is determined by the Poynting vector $\boldsymbol{S} = (\boldsymbol{E} \times \boldsymbol{B})/\mu_0$. The electromagnetic field far from the moving EV, can be expressed by the Eqs. (16) and (17). Thus, the Poynting vector can be calculated as

$$\boldsymbol{S} = \frac{q^2 |\boldsymbol{e}_r \times \dot{\boldsymbol{v}}|^2}{16\pi^2 \epsilon_0 c^3 r^2}\, \boldsymbol{e}_r = \frac{q^2 \dot{v}^2 \sin^2\theta}{16\pi^2 \epsilon_0 c^3 r^2}\, \boldsymbol{e}_r\ , \tag{66}$$

where θ is the angle between the direction of the acceleration and the direction to the observation point. And the power transported across a small surface $\Delta \boldsymbol{s} = r^2\, d\Omega\, \boldsymbol{e}_r$ is $dP = \boldsymbol{S} \cdot \boldsymbol{ds}$ or

$$\frac{dP}{d\Omega} = \frac{q^2 \dot{v}^2 \sin^2\theta}{16\pi^2 \epsilon_0 c^3}\ , \tag{67}$$

where $d\Omega$ is the solid angle subtended by the surface ΔS. The total power emitted into all directions is given by

$$P = \int_0^{2\pi}\int_0^{\pi} \frac{dP}{d\Omega} \sin\theta\, d\theta\, d\phi = \frac{q^2 \dot{v}^2}{6\pi \epsilon_0 c^3}, \tag{68}$$

which is known as *Larmor's formula.*

Let's estimate the power of radiation of the EV. Substituting the Eq. (22) for $\dot{v}$ and the Eq. (2) for q to Eq. (68), we have electromagnetic radiation power $P = 0.13$ watt.

It is well known that an electron gyrating in a magnetic field $B_0\, \boldsymbol{e}_z$ emits cyclotron radiation. How is this spontaneous emission affected by the electron motion in the crossed electric and magnetic fields, like $E_r(r)\, \boldsymbol{e}_r$ and $B_0 \boldsymbol{e}_z$ in the EV? Davidson et al. [16] have shown that in the crossed $E_r(r)\, \boldsymbol{e}_r$ and $B_0 \boldsymbol{e}_z$ fields the orbit of the electron perpendicular to $\boldsymbol{e}_z$ in the laboratory frame will become *biharmonic*, with two rotational frequencies ω^+_{re} and ω^-_{re} defined in Eq. (61). The energy radiated per unit frequency interval per unit solid angle is given by [16]

$$\eta(\omega) \equiv \frac{1}{T}\frac{d^2 I}{d\omega\, d\Omega} = \frac{e^2 \omega^2 T}{8\pi^2 c^3}\left\{ \frac{\sin^2(\Omega^+ T/2)}{(\Omega^+ T/2)^2}\, \frac{\omega_{re}^{+2}}{(\omega_{re}^+ - \omega_{re}^-)^2}\, (v_\perp^2 + \omega_{re}^{-2} r^2) \right.$$

$$+ \frac{\sin^2(\Omega^- T/2)}{(\Omega^- T/2)^2} \frac{\omega_{re}^{-2}}{(\omega_{re}^+ - \omega_{re}^-)^2} (v_\perp^2 + \omega_{re}^{+2} r^2)$$

$$- \frac{2\,\omega_{re}^+ \omega_{re}^-}{(\omega_{re}^+ - \omega_{re}^-)^2} \frac{\sin(\Omega^+ T/2)\sin(\Omega^- T/2)}{(\Omega^+ T/2)(\Omega^- T/2)}$$

$$\times \left[(v_\perp^2 + \omega_{re}^+ \omega_{re}^- r^2) \cos\left(\frac{(\Omega^+ - \Omega^-)\,T}{2} \right) \right] \Bigg\} , \tag{69}$$

where $T = L/v_z$ is the length of time that the electron is in the interaction region; k_z is the wave number and $\Omega^\pm$ is defined by

$$\Omega^\pm \equiv \omega - k_z u_z - \omega^\pm_{re} . \tag{70}$$

We see that the electron cyclotron emission spectrum $\eta(\omega)$ has two maxima located at $\Omega^+ = 0$ and $\Omega^- = 0$, or equivalently

$$\omega - k_z v_z \simeq \omega_{re}^\pm = \frac{1}{2}\omega_{ce}\left[1 \pm (1 - \frac{2\,\omega_{pe}^2}{\omega_{ce}^2})^{1/2} \right] . \tag{71}$$

For $2\omega_{pe}^2 / \omega_{ce}^2 \to 0$, the Eq. (71) gives $\omega^+_{re} \to \omega_{ce}$ and $\omega^-_{re} \to 0$. As the $2\,\omega^2_{pe} / \omega^2_{ce}$ increase, ω^+_{re} shifts downward and ω^-_{re} shifts upward. Therefore, from measuring the cyclotron emission from the EV, we may determine the parameter $2\,\omega_{pe}^2 / \omega_{ce}^2$ or n/B_o^2. If we know *n (or B_o)*, then we can determine the value of B_o *(or n)* for the EV.

H. CONCLUSIONS

In this paper, we have analyzed the macroscopic equilibrium properties for a moving electron cluster (EV) based on the electron plasma fluid description. We have shown that the self-consistent equilibrium is possible only in the toroidal system, that is the *EV is a toroidal electron vortex.*

We further deduced a necessary condition or criteria in which an EV could exist. The criteria indicate that the EV could exist at various combinations of the electron density, the directional velocity (or energy), and the size of the EV.

From the theory and results shown in this paper, we could imagine the EV process as follows: A large number of electrons are locally emitted in a very short time, $< 1/\omega_{pe}$. In the same time scale, a directional motion of a cluster of electrons are produced by a directional electric field. At the formation, the shape of the electron cluster may be spherical, ellipsoidal or spheroidal but will almost instantaneously be adjusted to a toroidal shape by the force balance requirement (Eq. (55)). A toroidal EV satisfying the criteria (Eq. (63)) will be formed. During the subsequent motion of the EV, along with the increase of the directional velocity, the electron density and the size of the EV will be changed or a large EV could be broken up into two or more smaller EV's in accordance with the equilibrium criteria, or destroyed by instabilities.

As a non-neutral electron plasma, it is expected that the various electrostatic and electromagnetic instabilities will occur in the EV. Therefore, an EV is not expected to exist in a field-free region. The presence of some dielectric interface, low density plasma, or wave-guide support etc. will greatly enhance the stability of the EV. These important problems are the subject for a future paper.

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